

# Into the twilight zone: how does WIPL–D perform in quasistatics?

Ari Sihvola<sup>1</sup>, Tapan K. Sarkar<sup>2</sup>, Branko Kolundžija<sup>3</sup>

<sup>1</sup>Electromagnetics Laboratory, Helsinki University of Technology  
P.O. Box 3000, FIN-02015 HUT, Finland

<sup>2</sup>College of Engineering and Computer Science, Syracuse University  
121 Link Hall, Syracuse, New York 13244–1240, USA

<sup>3</sup>Faculty of Electrical Engineering, University of Belgrade  
Bulevar kralja Aleksandra 73, P.O. Box 35–54, 11120 Belgrade, Yugoslavia

## Abstract

We focus on the question how well WIPL–D, a numerical code designed to tackle dynamical electromagnetic problems, can be applied to solving very low-frequency problems. In particular, the problem of the static polarizability of a dielectric sphere is calculated. This is done by enumerating the monostatic radar cross section of the object and taking the low-frequency limit. Peeling away the strong frequency dependence of the radar cross section, the remaining coefficient is proportional to the square of the static polarizability. The results show that there is around two decades of frequency range where the code works well and the situation is clearly in the quasistatic regime. In the example of a sphere of one-meter radius and relative permittivity 10, the low-frequency breakdown happens at around 10 kHz.

## 1 Introduction

Our aim is to study whether WIPL–D [1, 2] which is a code to solve electromagnetic problems with time-harmonic fields and finite wavelengths can be “misused” to calculate problems that are basically electrostatic. This is done bluntly by setting the problem with certain physical and geometrical dimensions and then progressively lowering the frequency of the electromagnetic field. This means that we are travelling from the dynamic regime into the statics. On this road, we are passing through the “twilight zone” (a striking and telling term by Weng Cho Chew, University of Illinois). At a certain point when going down, the wavelength is so many orders of magnitude larger than the dimension of the object that a full-wave code breaks down.

However, if we are already across the twilight zone when this breakdown happens, we are safe. This is because if the connection between the static and dynamic variables is known (they look different in these different regions) from the results we can already extract the desired parameters we were looking for in the first place.

We will show that this project is successful for the case of the polarizability of spherical dielectric objects (we presume that is working for other shapes as well) when it is calculated from the radar cross section using WIPL–D. The following presentation is a short resume of the results of our project, for more details, see [3]. The seminar which gave impetus to this project was organized at the Helsinki University of Technology in May 2004 [4].

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## 2 Radar cross section and polarizability

Radar Cross Section (RCS) carries the dimension of a surface, and is a measure how much an object scatters electromagnetic energy in the backscattering direction. This monostatic RCS is defined by [5]

$$\sigma_{\text{RCS}} = \lim_{r \rightarrow \infty} \frac{4\pi r^2 |E_s|^2}{|E_i|^2} \quad (1)$$

where  $E_s$  is the amplitude of the scattered field in the distance of  $r$  of the object in the backscattering direction when it is illuminated by a plane wave with electric field amplitude  $E_i$ .

On the other end of the frequency spectrum, in electrostatics, the concept of electric polarizability is extremely essential. The polarizability is the relation between the static dipole moment  $\mathbf{p}$  induced in an object and a static exciting field  $\mathbf{E}_{\text{st}}$ , being  $\mathbf{p} = \overline{\alpha} \cdot \mathbf{E}_{\text{st}}$ . In general, the polarizability  $\overline{\alpha}$  is a dyadic but for symmetric objects, like (isotropic and homogeneous) sphere or cube it is a multiple of the unit dyadic, equivalent to a scalar  $\alpha$ .

For a dielectric sphere, the polarizability is [6]

$$\alpha = 3V\epsilon_0 \frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} \quad (2)$$

where  $V$  is the volume and  $\epsilon = \epsilon_r \epsilon_0$  the absolute permittivity of the sphere. The polarizability is often given in the dimensionless normalized form  $\alpha_n = \alpha/(\epsilon_0 V)$ , and for the sphere it is

$$\alpha_n = \frac{\alpha}{\epsilon_0 V} = 3 \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad (3)$$

Note that in the conducting limit the normalized polarizability of a sphere is equal to 3.

In low frequencies, the scattering from a polarizable object can be calculated from the far field of a Hertzian dipole with the dynamic dipole moment amplitude  $j\omega \mathbf{p}$ , and the field amplitude in the backscattering direction is

$$|E_s| = \frac{\omega^2 \mu_0 p}{4\pi r} \quad (4)$$

Since the (static) dipole moment is connected with the incident field as  $p = \alpha E_i$ , we can write the connection between the low-frequency limit of the radar cross section and the static polarizability of the object as follows:

$$\alpha_n = \frac{\sqrt{4\pi\sigma_{\text{RCS}}}}{k_0^2 V} \quad (5)$$

where  $k_0^2 = \omega^2 \mu_0 \epsilon_0$  with the free-space parameters  $\epsilon_0, \mu_0$ .

## 3 Calculations and results

The monostatic radar cross section of a dielectric sphere was calculated with WIPL-D, more exactly, the parameter  $\sigma_{\text{RCS}}/\lambda^2$ . In the low-frequency limit, where Rayleigh scattering is dominant, this parameter has the dependency of  $\omega^6$ , in other words  $\lambda^{-6}$ . Therefore from the cross section in the low-frequency limit we can estimate the polarizability of the sphere.

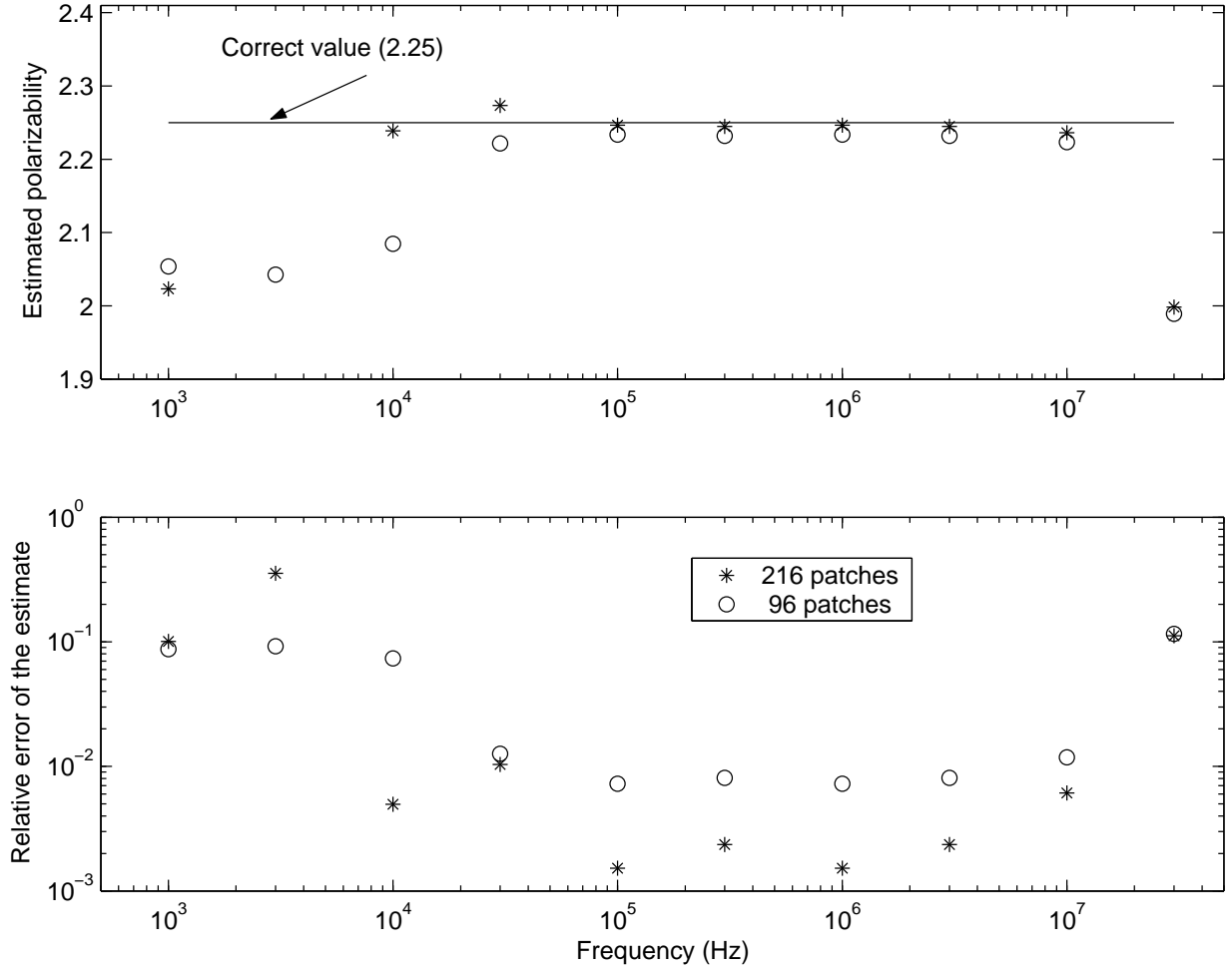


Figure 1: The estimate of the polarizability of a sphere with permittivity  $\epsilon_r = 10$ , calculated with WIPL-D in a broad frequency range. The radius of the sphere is 1 m. The sphere was approximated with two different plate-models, corresponding to 216 and 96 patches over the whole sphere. In the lower part of the figure is the relative error of the calculations.

The results of the WIPL-D calculations are shown in Figure 1. These are for a dielectric sphere with radius  $a = 1$  m and relative permittivity  $\epsilon_r = 10$ . The results shown are the normalized polarizability estimates which are calculated using the relation (5) and the RSC results from the software. The estimates also compared with the exact value for the normalized polarizability  $\alpha_n = 9/4$ .

In the calculations, we have used the equivalent radius for the sphere. As can be seen in Figure 2, the surface is an approximation for the sphere. The equivalent radius is derived from the condition that the maximal deviation of the approximate surface with respect to the perfect sphere is minimal [2].

The time to calculate one frequency point was 10 seconds for the denser sphere and 2 seconds for the more coarse quasisphere on a Dell Inspiron 8200 laptop machine with clock speed of 1 GHz. In the code, we used grade Enhanced 3, which is necessary for very low frequency calculations.

As we can see from the results in Figure 1, very good estimates for the static polarizability of an object can be found from the dynamic calculations with WIPL-D. The sphere calculations give with

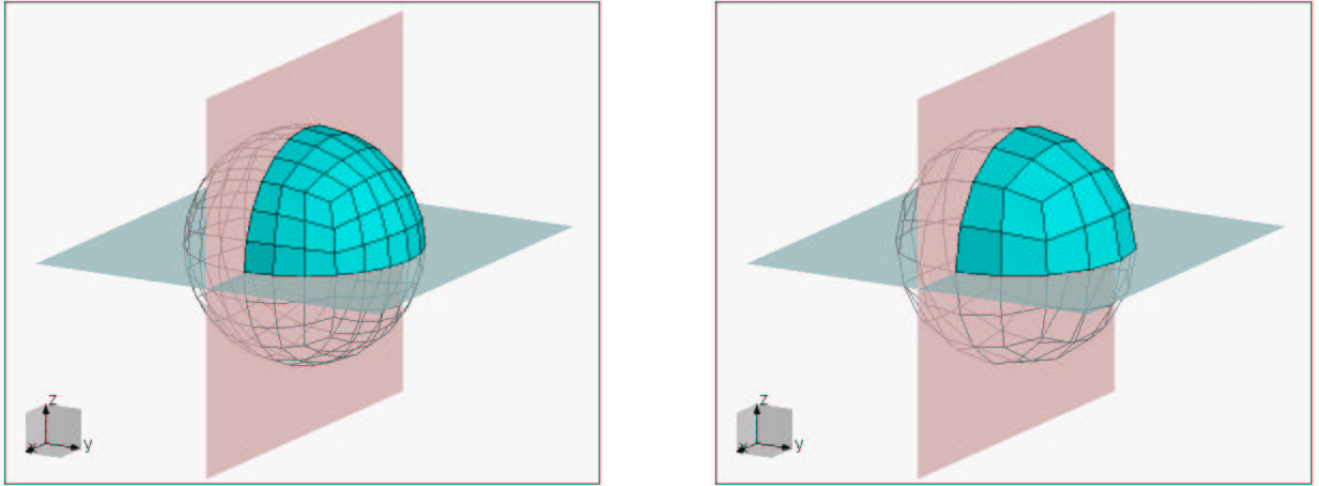


Figure 2: The densely and coarsely meshed sphere used in the WIPL-D calculations. Note the use of the two symmetry planes. The number of quadrilaterals is 54 and 24 in one quadrant, respectively.

reasonable calculation times an accuracy of three significant digits in  $\alpha_n$ , as long as we are around the proper frequency. It is important that the analysis is done in sufficiently low frequencies so that Rayleigh scattering dominates. In the figure we can clearly observe this high-frequency divergence starting at around 10 MHz. This makes sense because then the wavelength is 30 meters, and higher frequencies than that correspond to wavelengths which come close to the diameter of the sphere (2 meters).

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